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This paper describes means for self-organizing a nonrigid, distributed, transmit-receive antenna array for use in airborne radar. The techniques are applicable to ground-based or shipborne radar as well. Methods are described for initializing the array using various primary microwave illuminators. The description of phase conjugation techniques and means for distributing phase reference to all elements in the array are the central parts of the paper.

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CONTENTS

PHASE SYNCHRONIZING
A NONRIGID, DISTRIBUTED, TRANSMIT-RECEIVE RADAR ANTENNA ARRAY

## 1. INTRODUCTION

An airborne radar with a phased array the size of the aircraft would have many desirable attributes: For fixed transmitter power, the large aperture would provide unusually large detection range. Second, for a given desired performance, the transmitter power could be reduced dramatically [1]. Next, the small horizontal beamwidth would offer a resolving power comparable to human vision. Lastly, adaptive interference cancellation circuits operating from the large aperture would suppress jamming very close to the beam axis [2].

An aircraft-size array would consist of flush-mounted antenna elements distributed throughout the skin of the aircraft. Structural members, doors, windows, etc., would preclude a regular distribution of element locations. Furthermore, the nonrigidity of the airframe and skin would displace the elements from their design positions when in flight. Thus the design principles must be based upon the properties of the random array [3], [4] and self-cohering or adaptive beamforming techniques must be used to compensate for the time-varying positions of the array elements [5], [6]. Such a system is called a radio camera.

Adaptive beamforming is a retrodirective process in which a beam is focused upon a synchronizing source external to the array. The synchronizing source for an airborne radio camera must be another aircraft, a surface target or clutter [7, Chapter 11].

An airborne radio camera can use a conventionally designed transmitter and a distributed receiving array. Alternately, transmission as well as reception can take place through the self-cohered array. The latter is a much more formidable problem. Means for accomplishing it is the subject of this paper.

There are two reasons for accepting the increased complexity of the transmit-receive system. First, as shown in [1], the transmit-receive system offers a huge performance advantage. Second, the sidelobes of the random array are high because of the random locations of the elements: The average sidelobe level is $N^{-1}$ (N = number of elements) [3] and the peak sidelobe level is about 10dB higher [4]. By transmitting through the same array the side radiation pattern is squared, ASL drops to $N^{-2}$ and PSL $\approx 100N^{-2}$.

There are five distinct matters to be addressed in the design of an adaptive beamforming and scanning system which both transmits and receives through the self-cohered array:

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- initial illumination for phase cohering
- the phase synchronizing source
- phase conjugation
- generation and distribution of a phase reference
- the nature of the transmitter

Discussions of these topics constitute the body of the paper.

## 2. INITIALIZATION OF THE ARRAY

The large adaptive array must first self-cohere as a receiving system on the radiation from a synchronizing source. The synchronizing wave may be either the radiation from an active beacon or the reradiation of a large reflector illuminated by a microwave transmitter. Following phase synchronization-upon-reception, the system must organize the array as a transmitter and focus it retrodirectively upon the source.

Achievement of this objective is not a simple task. There is a logical difficulty in its accomplishment plus a host of practical obstacles. The logical difficulty is the following: The system requires both a radiator to illuminate the target as well as a large receiving aperture. The large size of the receiving aperture implies a priori unknown geometric distortion in the array which the self-cohering process corrects. The correction is an electrical compensation in the receiving system and is not a physical or geometric correction of the distorted array; therefore, the array is not suitable for transmission prior to adaptive beamforming. But adaptive beamforming cannot be accomplished prior to illumination of the target by the transmitter. Yet without transmission there is no target illumination and therefore no adaptive beamforming and imaging.

To solve the problem the adaptive beamforming system must be initialized prior to using the array as a spatially coherent transmitter. Phase synchronized narrow-beam radiation may then take place through it. Thus there are two separate processes which must be accomplished. The terms acquisition and tracking may be applied to them, reminiscent of the target acquisition and tracking functions performed by radar systems. Another term for acquisition is initialization. To accomplish acquisition the array must react to target reflections from a radiation field which may or may not be produced by the same array. The self-cohering process operates on these target reflections to phase synchronize the system. After initial synchronization the same array is then used for coherent transmission. Subsequent reflections from the target keep the array synchronized.

The problem, then, falls into two parts, initialization and tracking. The solutions, or rather the approaches to the solutions, are different in kind. The tracking problem is primarily a technical one while the solution to the acquisition problem is also influenced by the nature of the system and the constraints upon it. The tracking problem is relatively easy to solve once phase synchronization of the transmitter has been accomplished: When the transmitter beam is cohered, the receiving array need only follow the rules described in [6], and the transmitting array has to be updated from time to time. This paper deals with the more difficult of the two problems, that of acquisition or initial phase synchronization of the array.

## 3. SENSITIVITY TO SYSTEM CONFIGURATION

Figures 1 and 2 illustrate how the system configuration can impose limitations or constraints upon the acquisition method. In the first figure an array of N independently positioned elements is shown. Each element has radiating area $A_e$ and gain

$$G_e = \frac{4\pi A_e}{\lambda^2} \tag{1}$$

The total aperture area

$$A = N A_e \tag{2}$$

The available transmitting power P is divided equally among the N elements which are each driven by $P_e$ watts; thus

$$P = N P_e \tag{3}$$

Now let it be desired to self-cohere on a target of radar cross-section $\sigma_T$ at a range R, and let it be assumed that the total power P available for transmission is sufficient to do so provided that the array was initially synchronized. In this case the power P transmitted through aperture A and reflected from the target returns a signal to each receiving module strong enough to permit adaptive beamforming. That signal strength is easily calculated. The effective radiated power from the synchronized array is

$$PG = \frac{4\pi PA}{\lambda^2} \tag{4}$$

At the target distance the power density

$$W = \frac{PG}{4\pi R^2} = \frac{PA}{\lambda^2 R^2} \tag{5}$$

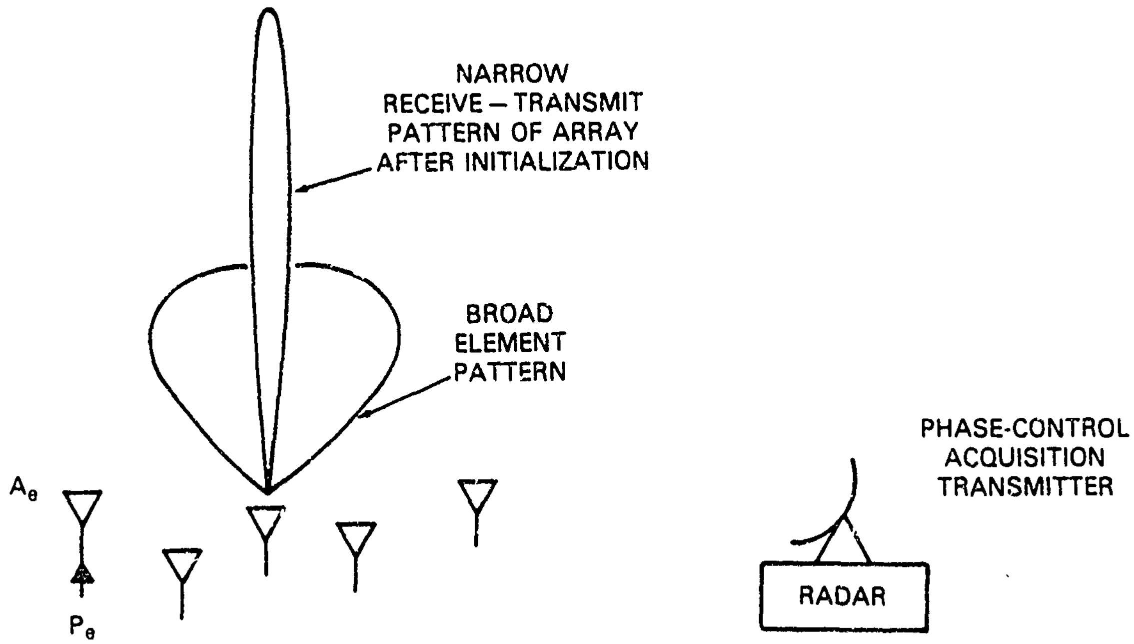



FIGURE 1. ARRAY OF N INDEPENDENTLY POSITIONED ELEMENTS

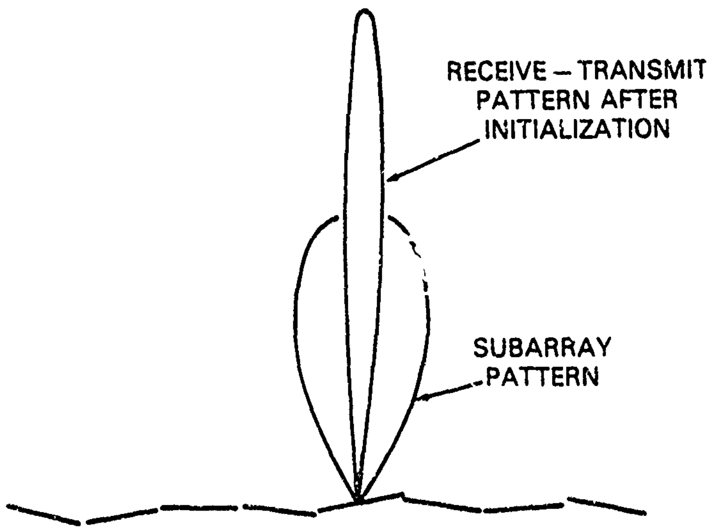



FIGURE 2. M EDGE-CONNECTED RIGID SUBARRAYS EACH HAVING $\frac{N}{M}$ ELEMENTS

The reradiated power in the direction of the array is (5) times $\sigma_T$, which, when divided by $4\pi R^2$, is the reflected power density at the array. The signal power S received by a single antenna module is the power density times the element area. Thus

$$S = \frac{N^2 P_e A_e^2 \sigma_T}{4\pi\lambda^2 R^4} \tag{6}$$

which is assumed to be sufficient to allow self-cohering. However, prior to phase synchronization the individual transmitters do not radiate coherently. Hence the effective radiated power is less than (4) and the received power per element is less than (6). The difference may be several tens of decibels. The system could be overdesigned to provide this margin, but then it would be overpriced for its function.

Therein lies the difficulty, for initialization of the system does require considerable power margin. One way to try to beamform the array in Figure 1 is to radiate $P_e$ , the available power per element, through a single element. The received signal power at each element

$$S_1 = \frac{P_e A_e^2 \sigma_T}{4\pi\lambda^2 R^4} = \frac{S}{N^2} \tag{7}$$

is a factor of $N^2$ smaller than S. Since N is likely to be the order of $10^3$ or larger, the received signal power can be 60 dB or more below the level necessary.

This sad situation can be alleviated by a factor of N by either of two methods. The first is to radiate the total power $NP_e$ from a single element, which requires that the element be capable of handling the total power. Since N will be the order of $10^3$, a separate high power transmitting element will most often be practical. The second procedure radiates from all N elements even though they are not cophased. The average received signal power per element becomes

$$S_{NON} = \frac{N P_e A_e^2 \sigma_T}{4\pi\lambda^2 R^4} = \frac{S}{N} \tag{8}$$

That this is so is seen by examining the field strength at the target for the three conditions that lead to (6), (7), and (8). The effective radiated power from a single element is

$$P_e G_e = \frac{4\pi P_e A_e}{\lambda^2} \tag{9}$$

At the target distance the power density

$$W_1 = \frac{P_e G_e}{4\pi R^2} = \frac{P_e A_e}{\lambda^2 R^2} \tag{10}$$

and the complex strength of the electric field

$$E_1 = \sqrt{W_1 Z_o}\; e^{j\phi_1} \tag{11}$$

where $Z_o$ is the impedance of free space and

$$\phi_1 = (kd_1 + \phi_{o1}) \bmod 2\pi \tag{12}$$

In (12) $d_1$ is the distance from the element to the target and $\phi_{o1}$ is the total phase shift through the transmitter from the reference oscillator in the system to the antenna. Now let $P_e$ be radiated through each element. The total field strength is

$$E_N = \sqrt{W_1 Z_o} \sum_{i=1}^{N} e^{j\phi_i} \tag{13}$$

If the array were initially synchronized all $\phi_i$ would be equal. In that case

$$|E_N| = N|E_1| \tag{14}$$

and the power density at the target is

$$W = N^2 W_1 = \frac{N^2 P_e A_e}{\lambda^2 R^2} = \frac{PA}{\lambda^2 R^2} \tag{15}$$

which leads to (6). If the array is not synchronized upon transmission the $\phi_i$ are random and independent and the expected power density at the target is

$$E\left\{W_N\right\} = W_1 E\left\{\sum_i^N \sum_j^N e^{j(\phi_i - \phi_j)}\right\} = NW_1 \tag{16}$$

which leads to (8). Thus the synchronized transmitting array has a power gain N times larger than the same array prior to phase synchronization; the latter in turn provides a phase-synchronizing signal at the array N times larger than when only a single element is used for transmission. This reduces the required margin from the order of 60 dB to 30 dB.

Now let the array consist of M rigid subarrays, each having N/M elements. Figure 2 shows an edge-connected example of such an array. Assume that the subarray is small enough physically that its radiation pattern encompasses the entire sector to be scanned by the radio camera. The N/M elements in each subarray can be combined into a single coherent aperture of area

$$A_{sub} = \frac{NA_e}{M} \tag{17}$$

through which power

$$P_{sub} = \frac{N P_e}{M} \tag{18}$$

is radiated. This radiation from a single subarray would result in an element signal

$$S_{N/M} = \left(\frac{N}{M}\right)^2 \frac{P_e A_e^2 \sigma_T}{4\pi\lambda^2 R^2} = \frac{S}{M^2} \tag{19}$$

which is larger than (7) by the factor $(M/N)^2$. Noncoherent radiation from all M subarrays further raises this level by a factor of M to

$$S_{NON\ N/M} = \frac{S}{M} \tag{20}$$

These equations are summarized in Table 1 which demonstrates how strongly dependent is the available signal power for the adaptive-beamforming acquisition function upon the configuration of the system.

| TRANSMISSION | RELATIVE RECEIVED SIGNAL POWER FOR INITIAL ADAPTIVE BEAMFORMING | | |
|---|---|---|---|
| Single element | 1 | -60 dB | |
| N noncoherent | N | -30 | |
| N coherent | $N^2$ | 0 ← | Adaptive Beamforming Tracking Mode |
| $\frac{N}{M}$ coherent | $\left(\frac{N}{M}\right)^2$ | -40 | |
| $\frac{N}{M}$ coherent, M subarrays noncoherent | $\frac{N^2}{M}$ | -20 | |

TABLE 1 - SIGNAL POWER AVAILABLE FOR ADAPTIVE BEAMFORMING ACQUISITION FUNCTION DEPENDS UPON SYSTEM CONFIGURATION. IN LAST COLUMN N = 1000 ELEMENTS, M = 100 SUBARRAYS.

The upper two lines in the table pertain to Figure 1. At best the acquisition beamforming signal is 30 dB below the level of the phase control signal during tracking. The system designer has several alternatives for overcoming this deficiency. These are discussed in the following section.

## 4. PRIMARY OR INITIAL ILLUMINATOR

Phase coherent transmission is not available from the full array until after the array has been self-cohered. Many forms of initializing illuminators are possible, five of which are described below:

(1) The first is to overdesign the transmitter by the necessary margin of N, which is the relative gain of the synchronized array to the nonsynchronized array. This is a costly choice.

(2) Use a separate high gain illuminator for initial synchronization and transfer power to the array afterwards. Figure 1 shows a radar transmitter being used for this purpose. This is an excellent approach when an auxiliary transmitter is available.

The auxiliary radar transmitter might be a nearby radar operating in the same frequency band but not at the same frequency. The method is illustrated in Figure 3. The auxiliary transmitter radiates at frequency $\omega_o'$ to a large target of opportunity (TOO). Target reflections at that frequency phase synchronize the receiving system. Phase conjugation of the received waveforms results in a receiving beam focused at the source [5], [6], [8]. However, the transmission of the phase conjugated waves is at $\omega_o \neq \omega_o'$, which offsets the transmitting beam somewhat [8]. Instead of a passive source the auxiliary radar could actuate an active beacon which would radiate to the array at the design frequency $\omega_o$ to self-cohere the array. This avoids the angular offset or "squint."

(3) An auxiliary radar transmitter in a different frequency band also could be used to initiate a beacon signal or induce TOO reflections. No angular offset results if the beacon reradiates at $\omega_o$. Otherwise the beam-angle offset upon transmission is too large and must be prevented by special phase conjugation circuits of the type described in [8]. An important advantage is obtained in using a lower frequency for initial synchronization with a TOO for then the lobes of the reradiation pattern are widened by the frequency ratio, thereby easing a size-limitation tolerance on the synchronizing source [7, Chapter 11].

(4) A rigid subarray of the full system can be used for coherent transmission of a broad beam to deliver the initializing signal to the beacon or the TOO (Section 3).

(5) The entire array can be driven noncoherently prior to self-phasing, in which case the average power density is $N^{-1}$ times the full power density of the system after the array is synchronized (Section 3). Although this loss is large, it can be compensated through use of a beacon or by initializing on the reflected signal from a nearby target. The squint problem is avoided since synchronization is at the system frequency $\omega_o$. The method is effective with a TOO reflection because of the $R^{-4}$ dependence of received signal power on target distance. To overcome a 30 dB initializing disadvantage (N = 1000) the reference reflection must be located no further than 18% of the maximum distance at which it could be placed if the transmitters were cohered. This distance reduces to 10% when $N = 10^4$. This technique is appropriate for an airborne system using ground or sea clutter for the reference target [7, Chapter 10]. It is

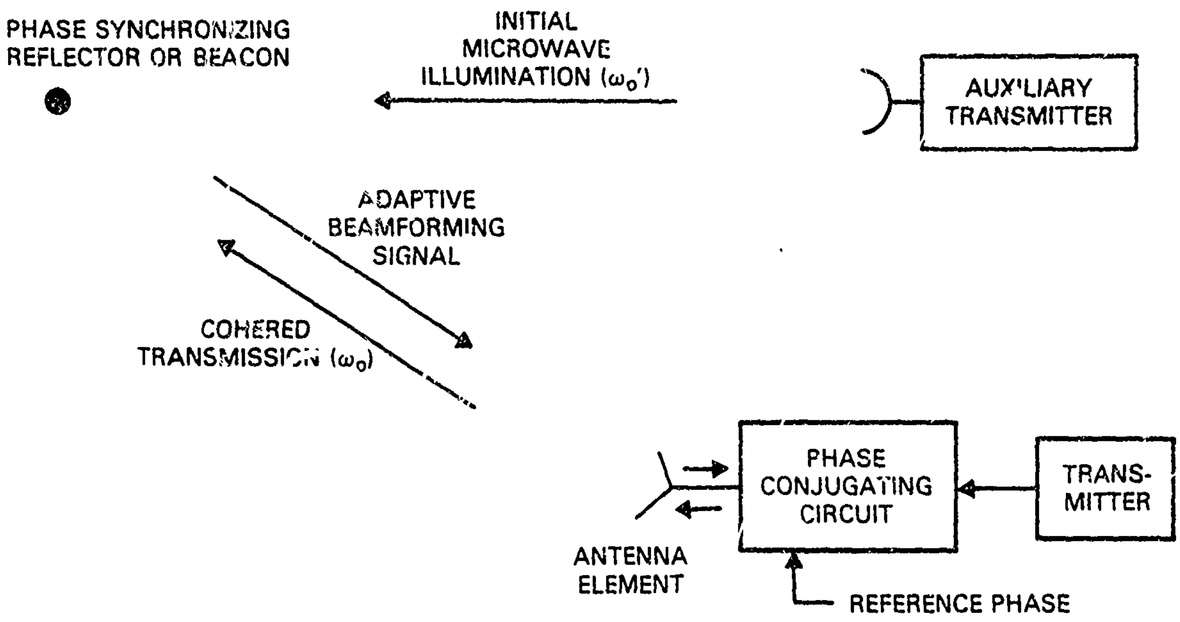



FIGURE 3. COMPONENTS OF ADAPTIVE BEAMFORMING TRANSMIT-RECEIVE ARRAY USING AN AUXILIARY TRANSMITTER FOR PHASE SYNCHRONIZATION. THE LOWER PART OF THE FIGURE SHOWS ONE MODULE OF THE LARGE ARRAY.

illustrated in Figure 4. Two modules of the array are shown. (Although the modules are shown in greater detail in this figure than in Figure 3, they are basically the same as the ones required for that figure.) Each consists of an antenna element, circulator (or any other diplexer), receiver, phase-stable reference oscillation common to the entire array, transmitter phase shifter, mixer, local oscillator cohered in frequency to the reference wave, and pulsed power amplifier.

To initialize the system each module radiates an RF pulse having common system frequency $\omega_o$ and random phase. The instantaneous transmission phase of the wave from the ith module may be designated $\omega_o t + \phi_i$. This wave arrives at the synchronizing source with delay $\omega_o t + \phi_i - \phi_{is}$ where $\phi_{is}$ is the phase delay from the module. The combined illumination at the reflector or beacon is $\sum_{i=1}^{N} a_i \exp j(\omega_o t + \phi_i - \phi_{is})$, the instantaneous phase of which is $\omega_o t + \Phi_s$, the subscript meaning source. The source wave is returned to the array with a different phase delay $\phi_{sj}$ at each module. Thus the signal phase received by the jth module is $\omega_o t + \Phi_s - \phi_{sj}$.

To focus the transmitting beam upon the source the radiation from the jth module must be changed from $\omega_o t + \Phi_s - \phi_{sj}$ to $\omega_o t + \phi_{sj} + \Phi_o$ where $\Phi_o$ is an arbitrary phase constant across the array. This is the phase conjugation process needed to achieve retrodirectivity. Methods for its accomplishment are discussed after the next section.

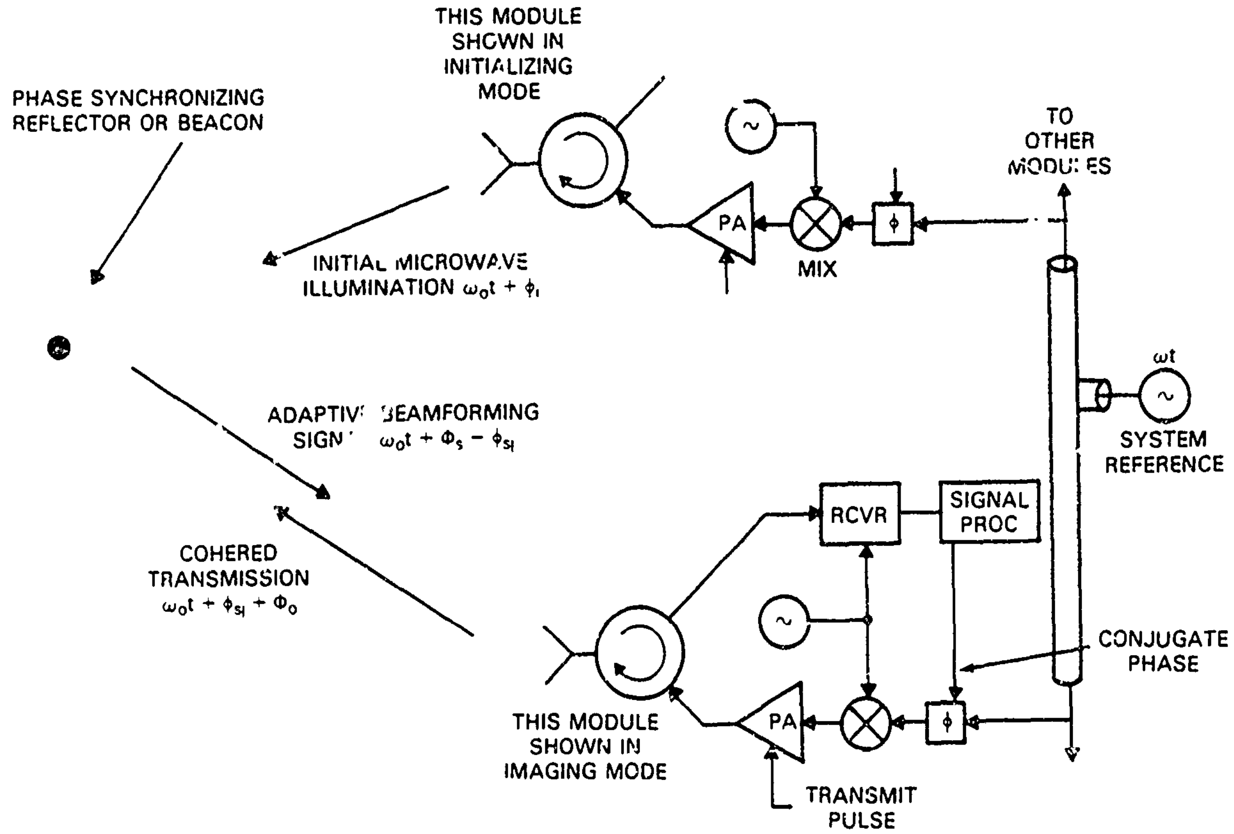



FIGURE 4. ADAPTIVE BEAMFORMING TRANSMIT-RECEIVE ARRAY WITHOUT AUXILIARY TRANSMITTER

Although many sources of initializing wave may be designed, all forms fall into two categories according to whether or not the initializing frequency $\omega_0'$ equals the system frequency $\omega_0$.

5. PHASE SYNCHRONIZING SIGNAL

The synchronizing source may be active or passive. A microwave transmitter is needed to initiate the synchronizing signal in either case, either by actuating an active beacon or by causing a strong reflection from a passive source. In principle a remote CW source also could be used. However, being on all the time it would make target detection more difficult. Its use is discussed in Section 11.

Unless the total available transmitting power is large enough to overcome the initial transmitting inefficiency, the power density of the initial illumination will be considerably lower than that of the full transmitting array after it is self-cohered. To compensate for this loss in signal strength, a passive synchronizing source must be larger or closer to the array than is necessary for a receive-only system. In Section 3 it was shown that the magnitude of the expected power loss is between $N^2$ and M (N = total number of array elements, M = number of rigid subarrays). These factors can be related as follows: Let P = total available transmitter power, G = effective transmitter antenna gain, $\sigma_T$ = radar cross section of the reference target, R = range to the reference target and S = received signal power.

Then

$$S \propto \frac{PG\sigma_T}{R^4} \tag{21}$$

Now let primed quantities pertain to the transmit-receive system prior to adaptive beamforming of the array, and let unprimed quantities pertain to adaptive beamforming with the receive-only array. The received signal powers S and S' needed to synchronize the systems may be assumed to be equal. Equating them leads to

$$\frac{PG\sigma_T}{R^4} = \frac{P'G'\sigma'_T}{(R')^4} \tag{22}$$

The gain ratio G/G' is between $N^2$ and M. Hence the relation between the various quantities becomes

$$M \lesssim \frac{P'\sigma'_T}{P\ \sigma_T}\left(\frac{R}{R'}\right)^4 \lesssim N^2 \tag{23}$$

In most design problems an auxiliary transmitter will not be available for initial synchronization of the array, in which case P' = P. This leaves only cross section and the range of the reference source as the available parameters. Both may be needed to overcome the loss in initial transmitting gain, which, from the example given in Table 1, was as much as 20 to 60 dB.

In some situations range to the reference source is not an available parameter as, for example, when the reference source is in the same range bin as the target. In this case, initial synchronization will take longer at best and may fail entirely. An alternate, initializing procedure, such as the use of a beacon, is recommended.

An equation equivalent to (23) for the beacon is easily written. $\sigma_T$ becomes replaced by beacon power $P_B$ and the range dependence drops from quartic to quadratic:

$$M \lesssim \frac{P'P'_B}{P\ P_B}\left(\frac{R}{R'}\right)^2 \lesssim N^2 \tag{24}$$

6. PHASE CONJUGATION TECHNIQUES

Retrodirectivity requires phase conjugation which in turn demands symmetry with respect of some reference. If $\phi_R\ (x;\theta_o)$ is the phase of the received wave at position x in the array due to a synchronizing source at angle $\theta_o$, and $\phi_T\ (x;\theta_o)$ is the transmitted phase variation needed to achieve retrodirectivity, the required relation is

$$\phi_T\ (x;\theta_o) = -\ \phi_R\ (x;\phi_o) \tag{25}$$

Equation (25) implies the existence of some reference phase from which $\phi_T$ and $\phi_R$ may be measured. The symmetry can exist in more than one domain. Spatial symmetry is utilized in one of the oldest forms of retrodirective array, the Van Atta array [9, 10]. Symmetrical sidebands in the frequency domain is another. A third is paired, symmetrical phase shifters.

The principle of the Van Atta array is illustrated in Figure 5. An array symmetrical about the center is required. Each element receives a signal, amplifies it and delivers it via a fixed length transmission line to its partner element on the other side of center, where it is rebroadcast. This arrangement satisfies the condition given in (25). No adaptive circuits are required yet the system is adaptive, i.e., it reradiates retrodirectivity without a priori knowledge of the direction of the arriving energy.

Dependence upon spatial symmetry is inappropriate for an airborne array distributed about the airframe. Here the array is assumed to be distorted, random and highly thinned. Hence some other form of symmetry must be used.

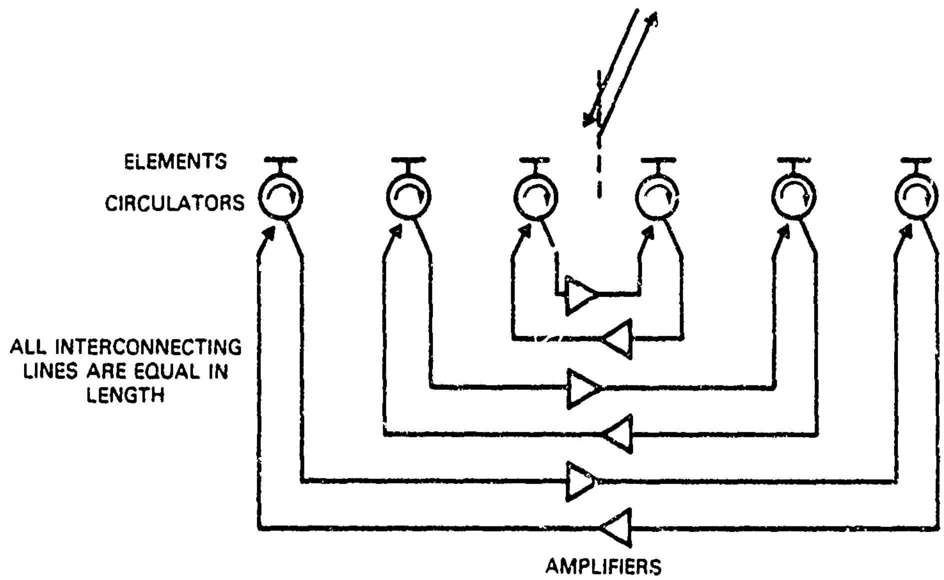



FIGURE 5. VAN ATTA ARRAY

The use of symmetrical sidebands often is practical. Let a signal characterized by the real (or imaginary) part of $\exp[j(\omega_o t + \phi)]$ be received and heterodyned (mixed) with a local oscillator at frequency $\omega_{LO}$. The mixer products are $\exp\left\{ j[(\omega_{LO} + \omega_o)t + \phi]\right\}$ and $\exp\left\{ j[(\omega_{LO} - \omega_o)t - \phi]\right\}$. The sidebands are symmetrically displaced about the local oscillator frequency and the lower sideband has the desired phase. Further, if the local oscillator frequency is made exactly twice the frequency of the received signal the lower sideband is $\exp[j(\omega_o t - \phi)]$.

This process is schematized in Figure 6. The input signal at frequency $\omega_o$ and phase $\phi$ (indicated by the instantaneous phase $\omega_o t + \phi$) is passed through a circulator where it mixes with the second harmonic at an arbitrary phase $\phi_o$. The difference frequency output of the mixer, at $\omega_o$, is passed by the band

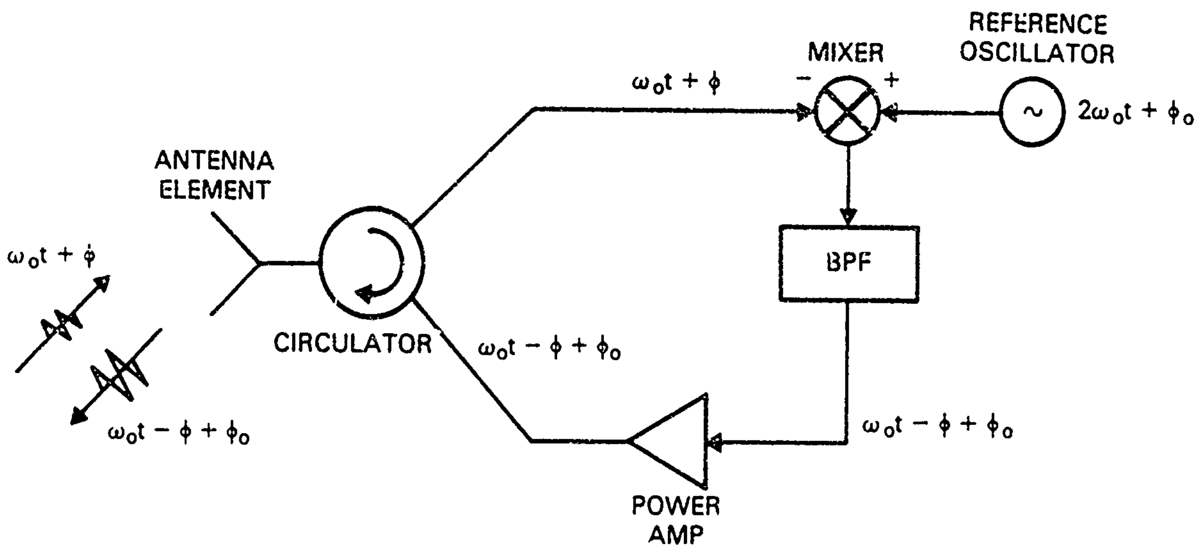



FIGURE 6. PHASE CONJUGATION SCHEMATIC

pass filter to the amplifier. The phase of this signal is $\omega_o t + \phi_o - \phi$. This signal is amplified and radiated. Provided that $\phi_o$ is constant across the array the radiation is retrodirective.

This circuit has two limitations. First, the down-converting mixer is a source of trouble because of the harmonic relation between the signals at its inputs. Being a nonlinear circuit the second harmonic of its fundamental frequency input will be generated. A current due to the second harmonic will flow in the source impedance of the second-harmonic input circuit, thereby altering the phase of the reference signal at $2\omega_o$. Also direct feedthrough of the input signal to the output will alter the phase of the output signal. Either the frequency of the reference signal must be different from $2\omega_o$ to avoid these troubles or the mixer must be carefully balanced so that neither second harmonic generation nor input-output leakage will effect the phase of the radiated wave.

The second problem is that this circuit transmits an amplified replica (with conjugated phase) of the received radar trace concurrent in time with the received signal. However, the desired transmission is an RF pulse (with conjugated phase) occurring at a later time. This means that the echo received from the synchronizing source must be sampled, its phase extracted, conjugated and applied to an oscillation at the echo frequency, which is gated at the appropriate time, amplified and radiated.

Figures 7-12 show several means for accomplishing this task. Each circuit is built from subcircuits which in turn can be organized in many different ways to affect phase-conjugated pulse transmission. These circuits are examples of many more that can be constructed. Each circuit and subcircuit has its own difficiencies which are more or less relevant according to the overall system of which the circuit is a part. Hence no one is absolutely better than another. The essential choices fall into two categories. The first is the method of storage of the phase information. It may be stored as the phase of a coherent oscillator, e.g., as in a phaselock loop (PLL). Or it may be stored as a digital number in the signal processor of the system. The second choice to be made is whether control of the phase shifter through which the signal to be transmitted is passed is open loop or closed loop. Other choices pertain to the components used, e.g., analog vs. digital phase shifters.

Figure 7 shows a means of phase conjugating when a digital signal processor is used to combine the signals received from the distributed array elements. The received signal is shown as an echo pulse having instantaneous phase $\omega_0 t + \phi$ . It is heterodyned to IF by a local oscillator at frequency $\omega_{LO}$ having some arbitrary phase $\alpha$. The LO is assumed to be cohered to the reference wave at the intermediate frequency $\omega$. The phase $\beta$ of the reference is assumed to be constant across the array. The IF pulse, with instantaneous phase $\omega t + \phi - \alpha$, is delivered to the signal processor. The signal processor measures the phase relative to the phase of some reference element in the array, whose phase is arbitrarily identified as zero phase. Since all signals entering the signal processor experience the same phase offset $\alpha$, the local oscillator phase cancels out. The negative of the measured signal phase $\phi$ is delivered as a control voltage to a voltage controlled phase shifter (VCPS) in the reference signal path. The output, having instantaneous phase $\omega t - \phi + \beta$, is up converted to form the transmitted wave. The transmitted phase is $\omega_0 t - \phi + \beta + \alpha$ .

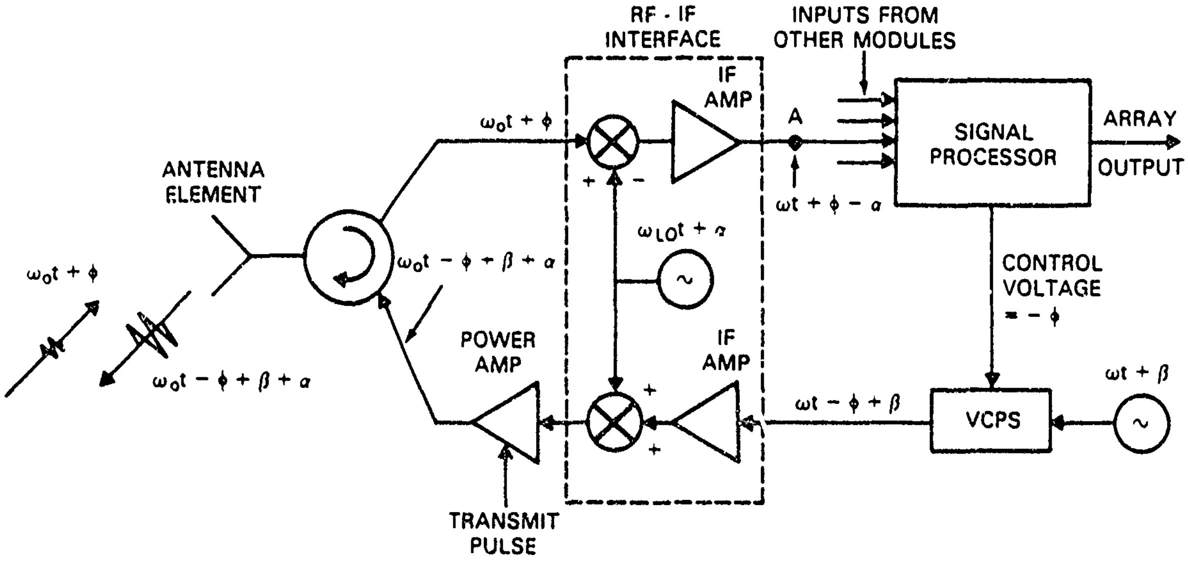



FIGURE 7. PHASE CONJUGATION CIRCUIT

The accuracy to which the signal processor measures $\phi$ (or $-\phi$) is influenced by noise and multipath. However the precision with which this measurement is made and held for delivery to the phase conjugation circuit can be made arbitrarily fine; it is determined by how many significant figures or bits are used in the measurement. Thus the quality of the delivered value $-\phi$ need be no poorer than that of the measured $+\phi$. Provided that $\alpha + \beta$ is a constant across the array the circuit of Figure 7 performs the desired operation.

Errors develop when the phase shifts through the system are not tuned out. Let the phase shift from the antenna, through the circulator, receiver and signal processor, to the VCPS be $\delta$ . The output of the phase shifter then becomes $\omega t-\phi-\delta+\beta$. Let the phase shift in the transmitting chain from VCPS, through the transmitter and circulator, to the antenna be $\eta$. The radiated wave becomes $\omega_0 t-\phi-\delta+\beta+\alpha+\eta$. Its phase is in error by $-\delta+\eta$. The variance of this error (in square radians) across the array, multiplied by $10 \log e \simeq 4.3$, is the expected loss in array gain in decibels [7, Chapter 13]. Thus each module must be carefully tuned to balance the phase shifts in the receiving and transmitting chains.

Another source of error is in the phase reference. Maintaining constant phase in the reference wave across the array is difficult. The variance of the phase of the reference wave also reduces transmitter gain according to the expression given above. This problem is common to all phase conjugation circuits and is discussed in Section 8. A system which avoids the necessity for a constant phase reference is described in Section 9. In addition, phase stability must be maintained in the relation between the LO and the reference wave. Cohering the LO to the reference via a frequency synthesizer is good practice.

The circuit of Figure 7 stores the signal phase in the signal processor and uses open-loop phase control. The next circuit (Figure 8) retains open-loop phase control but remembers the signal phase in a PLL. This circuit also demonstrates the use of paired, symmetrical phase shifters. The receiver chain is the same as in Figure 7 to point A, at which point the circuit branches. The received signal continues to the signal processor as before. It also is applied to the input port of the phase detector in a PLL in which the controlled element is a VCPS rather than the more common voltage-controlled oscillator (VCO). The control voltage in the loop drives the phase of the signal at the VCPS to be in quadrature with the input IF signal. As in the earlier system the signal through the VCPS is the reference oscillation at $\omega$ with arbitrary phase $\beta$. Hence the loop drives the VCPS to a phase shift $\phi - \alpha - \beta - \pi/2$. Ganged to the VCPS is a matched phase shifter with opposite phase. Its phase is $- \phi + \alpha + \beta + \pi/2$. While the phase of the VCPS is set in a closed loop, the paired phase shifter is set open loop. Carefully matched analog phase shifters are required; otherwise digital phase shifters must be used.

Digital phase shifters generally are preferable. The number of discrete phase-shift components required is easily calculated. If m is the number of quantization bits and M is the number of levels of quantization, the relation between them is given by $M = 2^m$. The loss in gain as a function of the number of quantization bits is given in Table 2. It is interesting to note that only a small number of quantization bits is needed to constrain the loss in array gain to a modest amount. Two bits limits the loss to less than 1 dB, three

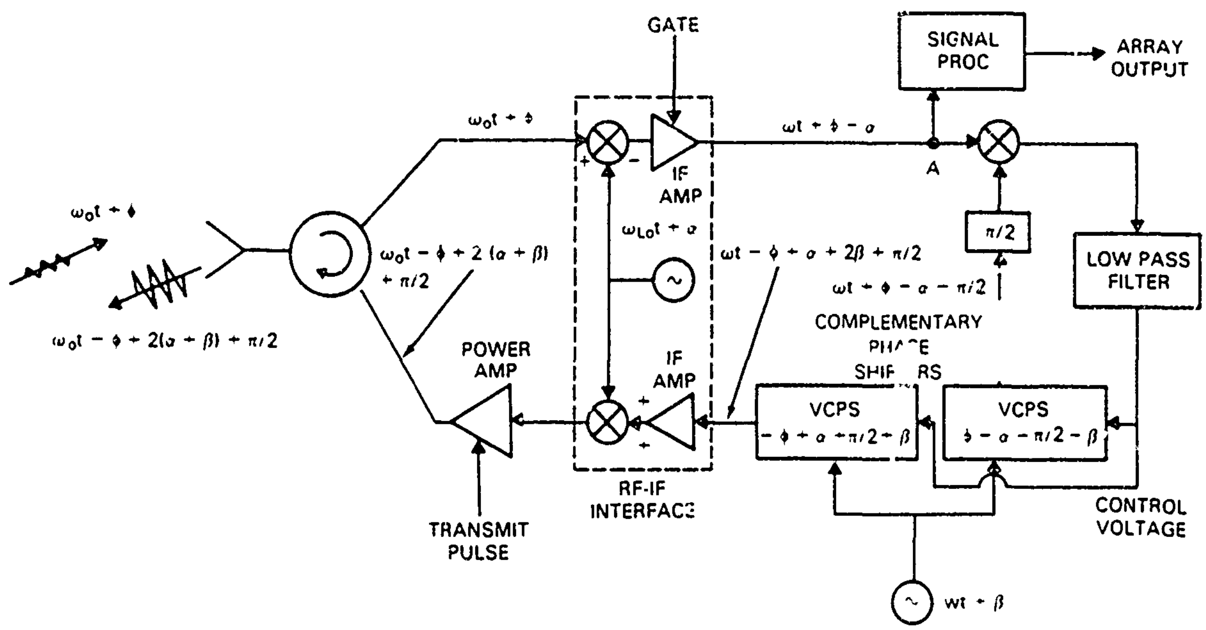



FIGURE 8. PHASE CONJUGATION CIRCUIT

bits to less than 1/4 dB. Two are insufficient as the 1 dB loss that would result would account for the entire tolerable loss in the system. As little as three, however, will be enough in many designs. The side radiation pattern also is affected by phase quantization errors, although the statistics in the sidelobe region are not. This is because the element positions are randomized, leading to Rayleigh sidelobe statistics. Further phase errors do not noticeably increase the sidelobe level; they only reduce the main-lobe gain.

Returning to the figure, it is seen that the input to the paired phase shifter is $\omega t + \beta$; its output, therefore, is $\omega t - \phi + \alpha + 2\beta + \pi/2$. This IF signal is heterodyned to RF via the mixer and the local oscillator at $\omega_{LO}t + \alpha$. The upper sideband at $\omega_0$ is selected by the power amplifier, providing a signal at the desired frequency having a phase $\omega_0 t - \phi + 2(\alpha + \beta) + \pi/2$. This signal is gated in the power amplifier by the transmit-pulse waveform from the radar synchronizer, and radiated. Provided that $\alpha + \beta$ is a constant across the array the high power transmitted pulse has the desired phase.

In addition to the cautionary comments pertaining to the previous circuit, this circuit has an additional problem: Because of the low duty cycle of the received echo pulse from the phase synchronizing TOO, it may not have sufficient energy to lock the PLL. The duty cycle in radar typically is $10^{-3}$. Hence the loop is driven only a miniscule fraction of the time. Fortunately the loop, being a narrowband circuit, provides considerable integration gain.*

*This circuit and the next lean heavily on the PLL. Texts [11-14] are recommended. In addition, an extensive 91-page bibliography of PLL literature is found in [15]. [16] deals specifically with the low duty cycle circuit.

TABLE 2 - EFFECT OF PHASE QUANTIZATION ON MAIN-BEAM GAIN (FROM [7], CHAPTER 13)

| Number of Quantization Bits $m$ | Number of Quantization Levels $2^m$ | Maximum Phase Error (rad) $\Delta\phi/2 = \pi/2^m$ | Main-Beam Gain Relative to Errorfree Array | Reduction of Array Gain (dB) $-10 \log_{10} G/G_o$ |
|---|---|---|---|---|
| 1 | 2 | 1.571 | 0.406 | 3.91 |
| 2 | 4 | 0.786 | 0.810 | 0.91 |
| 3 | 8 | 0.393 | 0.951 | 0.22 |
| 4 | 16 | 0.196 | 0.987 | 0.06 |
| 5 | 32 | 0.098 | 0.997 | 0.014 |

This may be insufficient, however. The situation can be improved if the array can be synchronized on range-extensive clutter, as is available to an airborne radar. In that case the receiver range gate can be widened by a factor of 10 to 100, with a corresponding increase in clutter energy.

Another aid is sweep integration, which unfortunately requires an additional circuit [17]. Its place is in the IF receiving chain at the input to the mixer of the PLL (marked A). Shown in Figure 9, the sweep integrator coherently adds several successive radar traces. The delay in the loop is exactly one interpulse period. Integration is with an exponentially decaying weighting function, as in an RC circuit. If the feedback voltage gain is $K \lesssim 1$, the effective number of echoes added coherently is $(1 - K)^{-1}$ and the time constant of the circuit is that number of interpulse periods. This number also is the increase in SNR, based on the reasonable assumption that the receiver noise is decorrelated from pulse to pulse.

In the remaining circuits the RF-IF heterodyning circuits are not drawn. No frequency shifting is shown but is implied. All nonfundamental components also are eliminated.

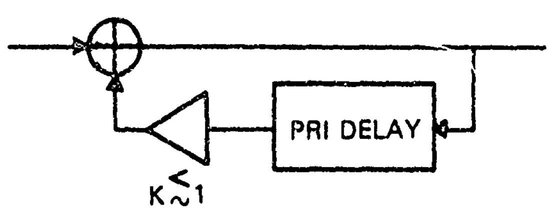



FIGURE 9. SWEEP INTEGRATOR

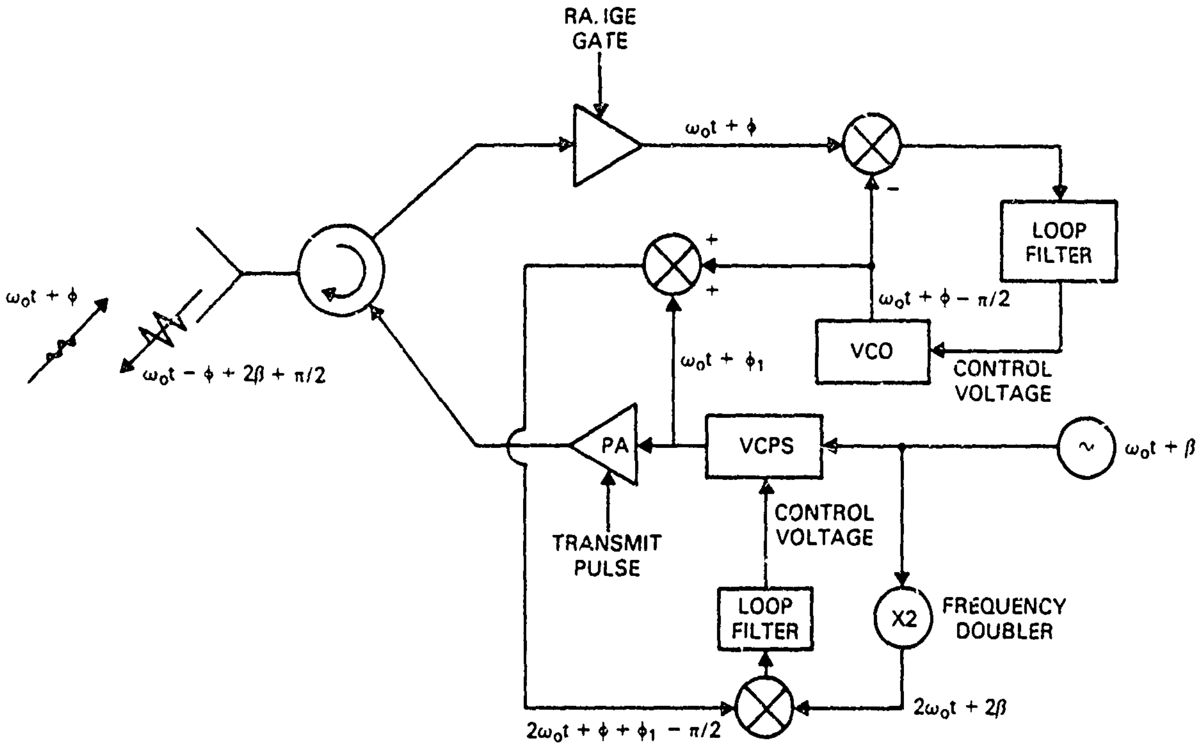



FIGURE 10. PHASE CONJUGATION CIRCUIT

Figure 10 shows a phase conjugating circuit which replaces the open-loop ganged phase shifter of Figure 8 by a phase shift circuit under closed-loop control.* The circuit is a PLL. The received TOO echo at $\omega_0 t + \phi$ is applied to the PLL. The VCO phase after the loop is locked is $\omega_0 t + \phi - \pi/2$. It is applied as one input to an up converter. The other input, also at $\omega_0$, is delivered by a voltage controlled phase shifter driven by the reference oscillation. Call its phase $\phi_1$. The up converter output is at the second harmonic frequency $2\omega_0$. Its instantaneous phase is $2\omega_0 t + \phi + \phi_1 - \pi/2$. Assume that adequate balance is achieved in the up converter to avoid second harmonic feed through. This signal is compared in a phase detector with the second harmonic of the reference. The beat product is integrated in the loop filter and applied as the control voltage to the VCPS. Convergence requires that $\phi + \phi_1 - \pi/2 = 2\beta$ or $\phi_1 = -\phi + 2\beta + \pi/2$. Thus the instantaneous phase of the CW output of the VCPS is $\omega_0 t - \phi + 2\beta + \pi/2$, which is the correct frequency and phase provided that $\beta$ is constant across the array. Hence the drive for the gated power amplifier is taken from this point.

* This circuit and those of Figures 11 and 12 were suggested by Prof. Y. Bar-Ness of Tel Aviv University.

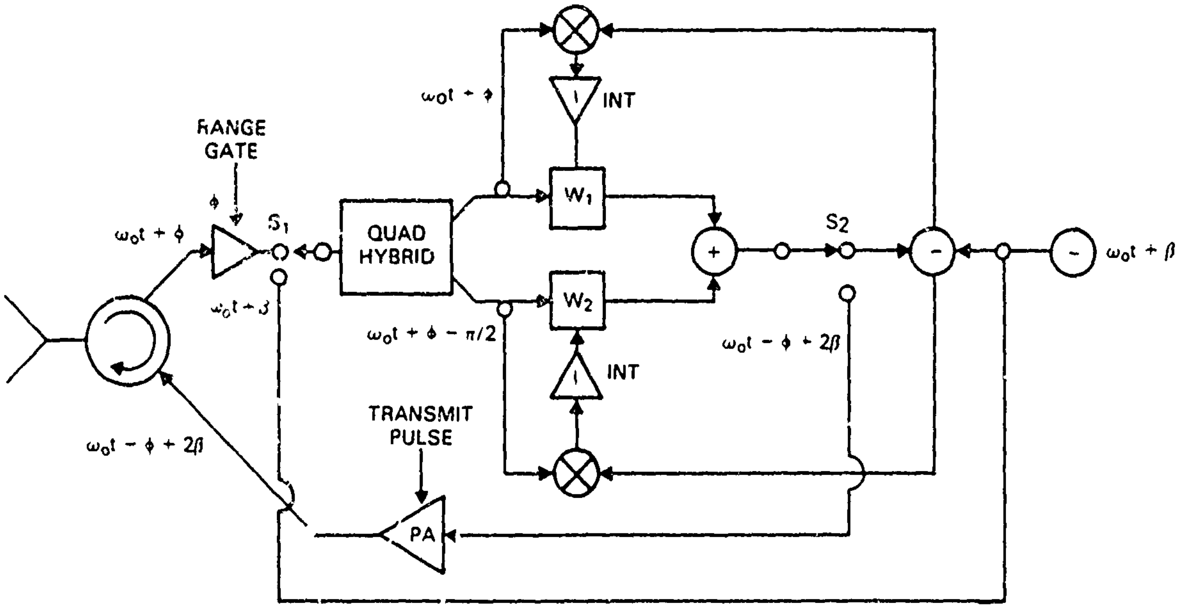



FIGURE 11. PHASE CONJUGATION CIRCUIT

The circuit of Figure 11 introduces another method of phase conjugation. The received echo pulse from the TOO is gated as before. The input to the phase conjugating network is switch $S_1$, connected as shown. The echo at $\omega_o t + \phi$ passes through a quadrature hybrid which delivers pulses at $\omega_o t + \phi$ and $\omega_o t + \phi - \pi/2$. These signals are weighted by real gain controls $w_1$ and $w_2$ and added. This sum, $w_1 \cos(\omega_o t + \phi) + w_2 \cos(\omega_o t + \phi - \pi/2)$, passes through switch $S_2$ to the comparator where it is subtracted from the reference wave $\cos(\omega_o t + \beta)$. The difference is fed back to the mixers of two correlators, the other inputs of which are driven by the quadrature outputs of the hybrid. The integrated mixer products drive the real weights $w_1$ and $w_2$ to those values that cause the sum waveform to equal $\cos(\omega_o t + \beta)$.

The portion of the circuit between the switches is used extensively in adaptive nulling and interference cancellation problems. The closed loops set the weights $w_1$ and $w_2$ so as to solve the equation

$$w_1 \cos(\omega_o t + \phi) + w_2 \cos(\omega_o t + \phi - \pi/2) = \cos(\omega_o t + \beta) \tag{26}$$

The solution is

$$\tan(\phi - \beta) = \frac{w_2}{w_1}, \quad w_1^2 + w_2^2 = 1 \tag{27}$$

When the loops have converged, the circuit between the two switches has transformed the input $\omega_0 t + \phi$ to the output $\omega_0 t + \beta$. In short the transfer function of the circuit at $\omega_0$ is

$$H(\omega_o) = e^{-j(\phi-\beta)} \tag{28}$$

which means that the circuit is a phase shifter having phase shift $\beta - \phi$. Following loop convergence the weights are frozen and both switches are thrown to their lower positions. The reference wave having phase $\omega_0 t + \beta$ then passes through the circuit and emerges with phase $\omega_0 t - \phi + 2\beta$. It is amplified, gated and radiated.

Figure 12 shows another version of the previous circuit in which the signals entering the comparator are hard limited in carefully matched limiters. Given that their amplitudes are matched it is only necessary to shift the phase of the input echo by $\beta - \phi$ to zero the comparator output. Only a single cancellation loop is needed as there is only a single parameter to be varied. A latching phase shifter, such as the digital phase shifter discussed earlier, is required. After loop convergence is completed the phase shift is frozen, the switches are thrown and $\omega_0 t - \phi + 2\beta$ is radiated.

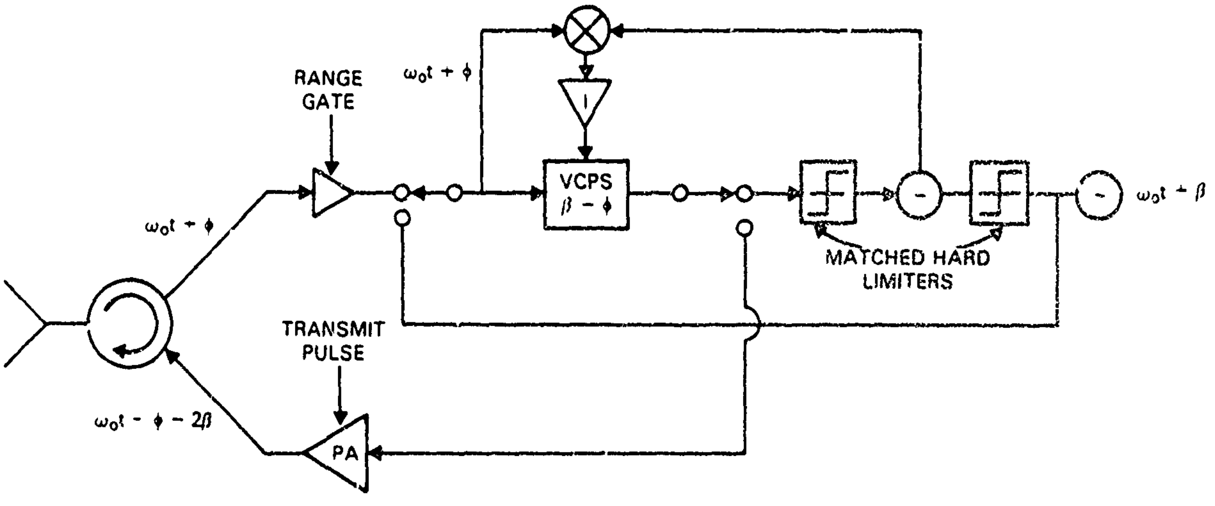



FIGURE 12. PHASE CONJUGATION CIRCUIT

## 7. ERRORS IN PHASE CONJUGATION

The idealized phase conjugation circuits described in the proceeding section are subject to phase errors. Two types predominate. The first is a random phase shift due to mistunings in open-loop portions of phase conjugating networks. The second is a linear phase shift due to a frequency offset. The former constitutes a random variation in phase across the array, the effect of which is loss in main-lobe gain as described earlier. Assuming a one dB total loss budget for the entire system, the allowed phase error is about ½ radian rms. It is evident that the allowed random error in phase conjugation is smaller still. Frequency offset, which has been ignored in the preceding section, may occur from two causes or reasons. First, the initializing microwave illuminator may be at a somewhat different frequency from the transmitting array. Second, the need for isolation between low-level incoming signals and high-level outgoing signals may force a frequency offset. The effects are the same in both cases.*

The magnitude of the frequency offset is determined by the manner in which the phase is conjugated. The simplest way is to adjust the phase shift at the initial frequency and accept the error which results. Let the reflecting source be at angle $\theta_o = \sin^{-1} u_o$ and the initializing (self-cohering on reception) frequency be $\omega' = k'c$. The phase of the wave across the array is $\phi_R = k'xu_o$ Let $i_o(x)$ describe the manner in which the transmitting array is excited. Let the conjugated phase be $-k'xu_o$ and let the wave number of the signal transmitted by the array be $k$. The radiation pattern becomes

$$\begin{aligned} f(ku;u_o) &= \int i_o(x) e^{-jk'xu_o} e^{jkxu} \, dx \\ &= \int i_o(x) e^{jkx(u - \frac{k'}{k} u_o)} \, dx \end{aligned} \tag{29}$$

Note that the beam no longer points to $u_o$ but to $k'u_o/k$. The error or displacement

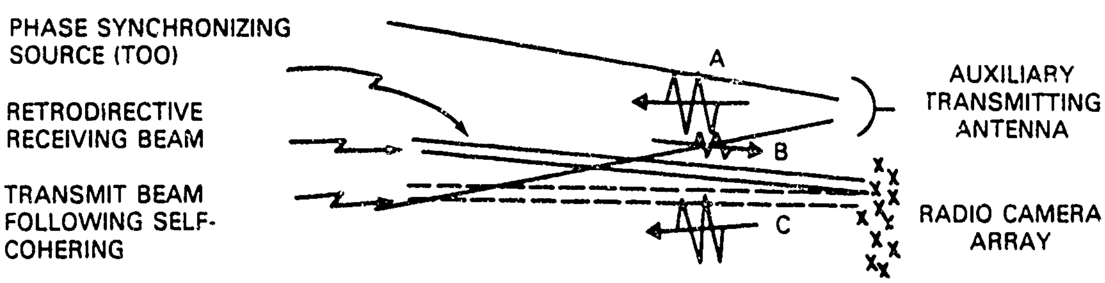
$$\Delta u_o = u_o \left(1 - \frac{k'}{k}\right) = u_o \left(1 - \frac{\omega'}{\omega}\right) \tag{30}$$

*Only the first of these two reasons for an offset frequency is pertinent to the airborne distributed array because after the array is phase synchronized it will radiate at exactly the frequency of the echoes (except for Doppler), as in ordinary radar. The second reason given above for a frequency offset pertains to the retrodirective array operating under CW or quasi-CW conditions, as is common in point-to-point communication systems. There it is possible for the system at each end of the propagation link to cohere its array to the signal radiated by the other. A frequency offset between the signals passing in the two directions is good engineering design so as to minimize the cross-talk and intermodulation between them. A frequency offset also is useful when a CW beacon is the reference signal. (See Section 11). The technique is described in [8].

is called the squint angle. Eq. (30) can be rewritten

$$\left|\frac{\Delta u_o}{u_o}\right| = \left|\frac{\Delta\omega}{\omega}\right| \qquad (31)$$

indicating that the magnitude of the fractional change in the beam-steering angle equals the magnitude of the fractional change in the frequency. The largest typical value of $\Delta\omega$ is the receiver bandwidth. Rarely will the angular displacement exceed one or a few percent of the scan angle. Such a scale distortion will be unimportant unless the beam displacement exceeds a beamwidth of the large array and no synchronizing source (e.g., a large TOO) resides within the transmitting beam. Figure 13 illustrates this unfortunate case. A phase conjugating circuit devoid of squint is required if this problem is anticipated [8].




A. RADIATION FROM AUXILIARY TRANSMITTER AT $\omega'$

B. WEAK ECHO FROM TOO AT $\omega'$

C. COHERENT TRANSMISSION FROM DISTORTED ARRAY AT $\omega$. TRANSMIT BEAM DISPLACED DUE TO FREQUENCY OFFSET.

FIGURE 13. BEAM SQUINT DUE TO FREQUENCY OFFSET

The reason why the angle distortion arises is that the phase is measured and conjugated at one frequency but radiation takes place at another frequency. The error is eliminated if the phase shift resulting from the conjugation process is correct at the new frequency. Then the phase of the radiation pattern of the retrodirective array, rewritten to exhibit explicit dependence on $\omega$ or $k$, is

$$f[k(u - u_o)] = \int i_o(x) e^{jkx(u - u_o)}\, dx \qquad (32)$$

The argument of the function is $k'(u - u_0)$. The beamsteering angle, therefore, is $u_0$. Hence the frequency change is no longer reflected in an angular displacement. The sole effect is a change in the angular scale, measured from $u_0$, by a factor $k'/k$. This scale change is of no consequence in adaptive beamforming.

## 8. PHASE REFERENCE

Each of the phase conjugation circuits of Section 6 requires a reference oscillation with constant phase $\omega t + \beta$ in every module in the array. This signal must be derived from an oscillator arbitrarily located in the array and delivered to each module by a circuit or subsystem. A frequency-stable and phase-stable oscillator is assumed as well as a frequency synthesizer capable of generating the local oscillator waveform.

Cables of equal and constant lengths can deliver the reference wave from the source to each module. This is a practical technique when the array is compact and the modules are contiguous. It becomes impractical when the array is large and distributed. Furthermore, being an open-loop system, differential phase changes between cables due, for example, to temperature differences or mismatches at connections, are passed directly as phase errors to the modules.

Circuits have been devised to deliver the phase reference from source to module, or from module to module. The major impetus has been design work for the Solar Power Satellite, a remarkable concept for the delivery of solar-generated electrical power from geosynchronous orbit to earth [18][19]. Figure 14 illustrates one method [20]. It consists of two distinct circuits separated by a cable having arbitrary phase delay $\Delta$. The reference signal passes between circuits via this cable at frequency $\omega$. It is provided to each phase conjugating circuit at twice that frequency and at the common reference phase $\beta$. Thus in the left circuit of Figure 14 a reference source is at frequency $2\omega$ and phase $\beta$.

The upper left circuit is a PLL. Its VCO phase is $\omega t + \phi_0$ where, for the moment, $\phi_0$ is an arbitrary value. Oscillator output is taken from the loop and passed, via the first diplexer (shown as a circulator), to the cable, which delivers $\omega t + \phi_0 - \Delta$ to the right-hand circuit. There the cable-delivered signal is doubled in frequency to provide the reference $2\omega + 2\phi_0 - 2\Delta$ for the second module. Thus both modules are driven by the same reference frequency. It is shown below that they are also driven to the same phase.

Prior to frequency doubling in the second module, the amplified wave at $\omega t + \phi_0 - \Delta$ is fed back to the second diplexer and returned to the first module. Its phase is further retarded by the cable delay $\Delta$. It is passed by the circulator to the mixer where it is heterodyned by the 90° phased shifted reference wave. The lower sideband is selected by the bandpass filter (BPF). Its output is the input to the phase detector of the PLL. The instantaneous phase of this wave is $\omega t - \phi_0 + 2\Delta + \beta + \pi/2$. The other input is $\omega t + \phi_0$, delivered by the VCO. The low-pass filtered output of the phase detector is zero when the loop drives the phase difference between the inputs to 90°. Hence

$$2\phi_0 - 2\Delta - \beta = 0 \text{ or } \beta = 2\phi_0 - 2\Delta \tag{33}$$

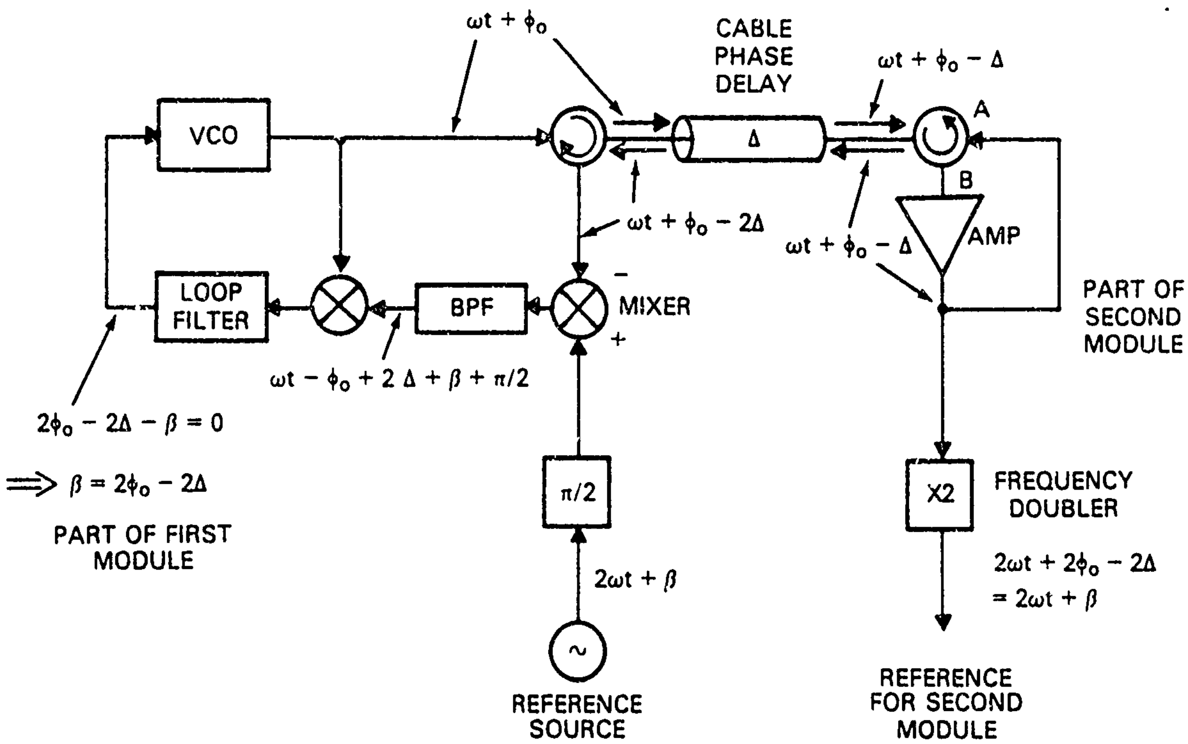



FIGURE 14. FREQUENCY AND PHASE-REFERENCE DELIVERY CIRCUIT (FROM [20])

which is the condition sought. Thus both modules have the same reference phase $2\omega t + \beta$ independent of the cable length between them. Other modules are fed in the same manner.

The circuit as drawn in Figure 14 is subject to several phase error sources. First, the nonlinear mixer will generate harmonics of the input signal. The second harmonic will add to the reference source at $2\omega$ to produce a net reference signal with altered phase. In addition, feed through the mixer at the fundamental frequency will alter the net phase of the signal delivered by the bandpass filter to the phase detector of the PLL.

A second source of phase errors is the phase shifts through all the non-closed-loop controlled portions of the circuit. The circulators, the bandpass filter and the signal-return loop are examples. This is a tuning-type problem common to all the preceding circuits as well.

Lastly, the signal-return loop has a special problem. Unless there is sufficient isolation in the circulator from ports A to B, the loop will oscillate. The amplifier is needed to overcome the signal losses in the cable in both directions. Hence the signal level delivered back to the circulator at port A is larger by the gain of the amplifier than the signal delivered by the circulator at port B. The ideal circulator (or other diplexer) provides zero coupling between ports A and B; the practical circulator has limited isolation. To avoid the danger of oscillation the isolation must exceed the amplifier gain which, in turn, must at least equal the two-way cable loss. Hence the maximum allowed cable loss is limited by the isolation available in practical circulators.

A small modification to the circuit avoids the more serious of these problems. Figure 15 shows the reference source frequency to be k times the VCO frequency and the frequency of the return signal to be $n\omega$. The VCO signal $\omega t + \phi_o$ again is delivered by cable to the next module, amplified, and returned. The return signal is frequency multiplied by the factor n, delayed by the cable, and mixed with the reference in the down converter. The phase of the output of the BPF is $(k-n)\omega t + \beta - n\phi_o + (n + 1)\Delta + \pi/2$.

The VCO output is multiplied in frequency by (k-n) to equate the frequency of the phase detector inputs. The loop drives these signals into quadrature, resulting in the phase equation

$$k\phi_o - \beta - (n + 1)\Delta = 0 \tag{34}$$

which implies that the instantaneous phase of the reference source

$$k\omega t + \beta = k\omega t + k\phi_o - (n + 1)\Delta \tag{35}$$

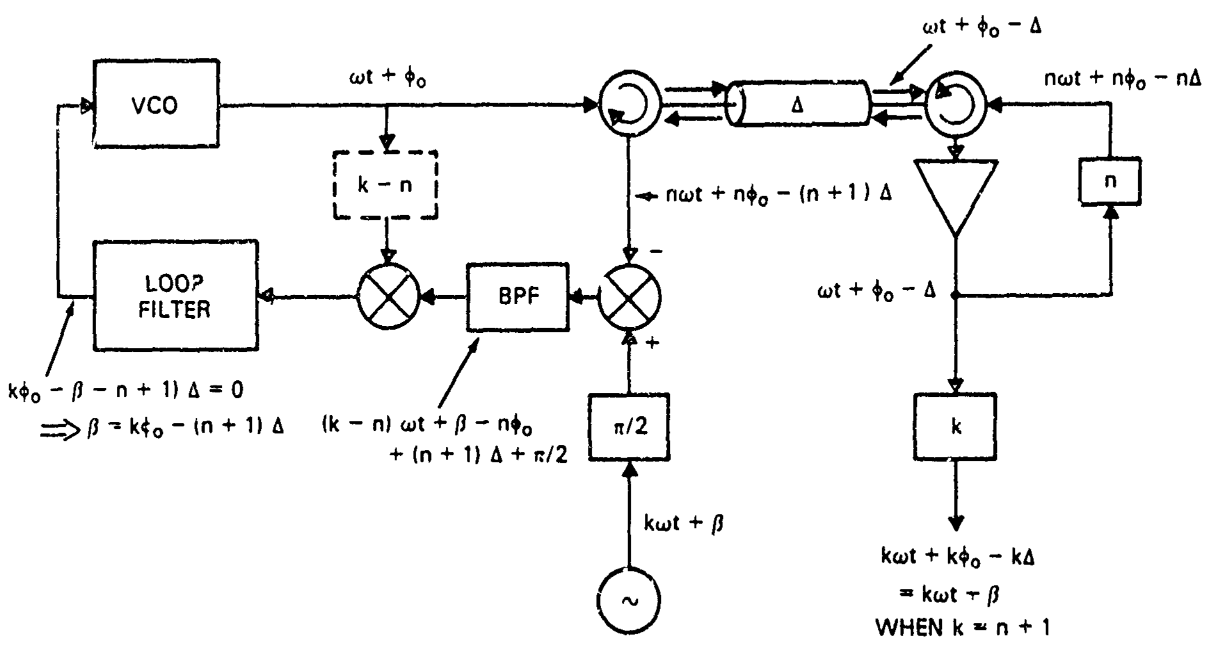



FIGURE 15. IMPROVED REFERENCE DELIVERY CIRCUIT

The output to the next module is derived from a frequency multiplication by the factor k of the signal delivered by the cable; its phase is

$$k\omega t + k\phi_o - k\Delta \tag{36}$$

The only condition required to equate (36) to (35) is

$$k = n + 1 \tag{37}$$

When this condition is met the desired phase reference is transferred from the first to the second module. In addition, (37) eliminates the need for the frequency multiplier (shown dashed) which follows the VCO and drives the phase detector; instead a direct connection may be made.

System frequencies are determined by the choice of a convenient VCO frequency $\omega$ and the desired frequency offset $\omega(n - 1)$. n must be small so that the bandwidths of the circulators and the delay time are not exceeded. n need not be integral; using modern frequency synthesizer techniques frequency multiplication by ratios of integers is easy to obtain. Thus n can be made close to unity. A value of 4/3 permits an adequate frequency separation between the input and the output of the mixer while not requiring excessive bandwidth of the components. The reference phase is delivered at $7\omega/3$ when n equals this value.

## 9. EXTERNAL CALIBRATION

Random, uncompensated phase shifts in components throughout the module and the reference-delivery circuit always will exist and will degrade main-lobe gain as described in Section 6. Periodic calibration or tuning is necessary to limit the loss in main-lobe gain to an acceptable level. Calibration can be manual or automatic in a small system but must be automatic in a system with a large array.

Figure 16 is a design using an external transceiver for system calibration.* It includes a procedure for correcting the phase of the reference wave at each module and thereby removes the need for a circuit of the type shown in Figure 15. In Figure 16 is one module of a distorted array along with the calibration transceiver. Each module has a phase-conjugating circuit (PCC) and a reference wave. Assume that the reference wave is stable in frequency and constant across the array but that the phase varies from module to module. Call the phase of the reference to the ith module $\beta_i$, or its total instantaneous phase $\omega t + \beta_i$.

The calibration unit is external to the array and in the general direction toward which the antenna elements are pointed. Its front end consists of a pulsed transmitter and receiver and is similar to the front end of an array module. The reference oscillation in the array is delivered to it either by

*This design is useful for shipboard and ground-based installations and is not appropriate for airborne use.

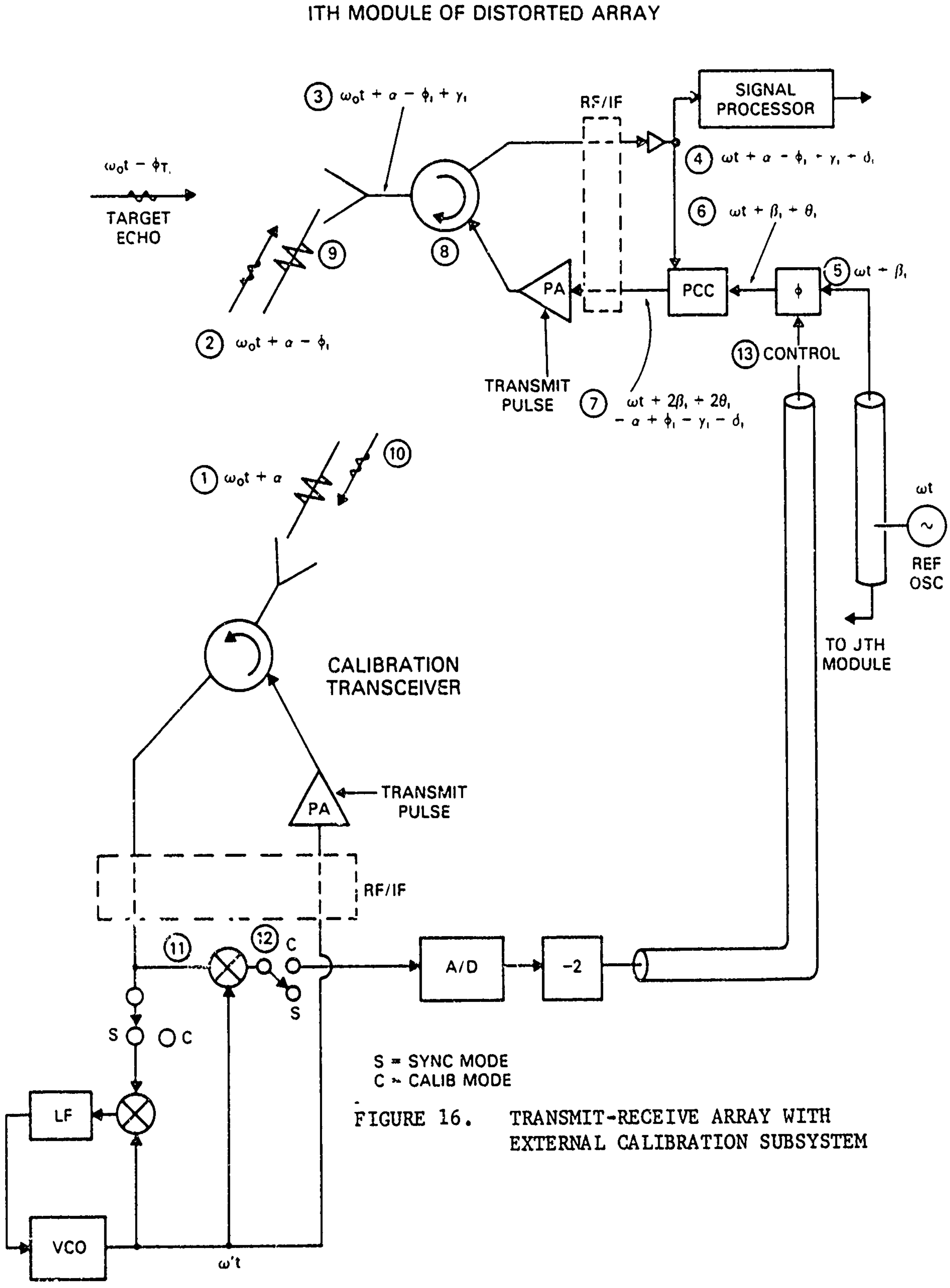



FIGURE 16. TRANSMIT-RECEIVE ARRAY WITH EXTERNAL CALIBRATION SUBSYSTEM

cable or by radio. The broadcast reference technique [21] is suitable and is the technique illustrated in Figure 16. When the calibration transmitter is connected in the SYNC mode (switches thrown to S), radiation of the system reference at some arbitrary phase from some arbitrary antenna within the array drives the PLL to a frequency $\omega'$. The phase $\omega' t$ of the VCO in the calibration transceiver is the reference phase of the entire system.

After the loop is locked the VCO output is heterodyned to RF and radiated. Its frequency is $\omega_o$, the radiation frequency of the entire system. Since there is some phase shift $\alpha$ from VCO to antenna, the radiated waveform is characterized by $\omega_o t + \alpha$, as indicated next to the symbol (1) in the figure. The system calibrates one module at a time. The system controller (i.e., the central computer) turns on each module in sequence. Upon arrival of the radiated signal (1) at the antenna of the ith module (2) the phase is $\omega_o t + \alpha - \phi_i$, where $\phi_i$ is the propagation phase delay. The phase shift through the antenna in the direction toward the calibration system is $\gamma_i$. Since the phase of the element pattern may be different in the direction to the target, $\gamma_i$ is explicitly retained in this description of system operation. Thus the signal phase after the antenna is $\omega_o t + \alpha - \phi_i + \gamma_i$. Similarly, there is a phase shift $\delta_i$ to the input (4) to the PCC, where the phase is

$$\text{Phase (4)} = \omega t + \alpha - \phi_i + \gamma_i + \delta_i \; . \tag{38}$$

The primary reference oscillator of the system runs freely at frequency $\omega$ delivering $\omega t + \beta_i$ to the input (5) to a digital phase shifter. The initial phase shift is some arbitrary value $\theta_i$, making the input reference phase (6) to the PCC $\omega t + \beta_i + \theta_i$. The output (7) of the PCC is

$$\begin{aligned} \text{Phase (7)} &= \omega t + \beta_i + \theta_i - [(\alpha - \phi_i + \gamma_i + \delta_i) - (\beta_i + \theta_i)] \\ &= \omega t + 2\beta_i + 2\theta_i - \alpha + \phi_i - \gamma_i - \delta_i \end{aligned} \tag{39}$$

After being heterodyned to RF and passed through the transmitter, the phase of the wave (8) is increased by $\eta_i$ to

$$\text{Phase (8)} = \omega t + 2\beta_i + 2\theta_i - \alpha + \phi_i - \gamma_i - \delta_i + \eta_i \tag{40}$$

$\gamma_i$ is added through the antenna (9) and the propagation delay $\phi_i$ is contributed upon arrival at the calibration system (10). There the instantaneous phase is

$$\text{Phase (10)} = \omega t + 2\beta_i + 2\theta_i - \alpha - \delta_i + \eta_i \tag{41}$$

It is evident that both the propagation delay and the antenna element phase shifts have dropped out.

This wave is received with the switches thrown to C, which places the calibration transceiver in the CALIB mode. The phase shift from antenna to phase detector (11) is $\mu$. Hence the phase detector output (12) is a video voltage proportional to

$$\text{Voltage (12)} = -2\beta_i - 2\theta_i + \alpha + \delta_i - \eta_i - \mu \tag{42}$$

This voltage is converted to a digital number, divided by two, and delivered (13) to the digital phase shifter in the module where it changes the phase through that component by

$$\text{Phase shift (13)} = -\beta_i - \theta_i + (\tfrac{1}{2})(\alpha + \delta_i - \eta_i - \mu) \tag{43}$$

The signal re-entering the PCC as a phase reference, therefore, is

$$\text{Reference Phase} = \omega t + (\tfrac{1}{2})(\alpha + \delta_i - \eta_i - \mu). \tag{44}$$

This is the desired reference phase: The random initial phase $\theta_i$ of the phase shifter and the random reference oscillation phase $\beta_i$ have been removed, the circuit phase shifts $\delta_i$ and $\eta_i$ cancel out during system operation (see below) and the remainder is a constant across the array.

Now let a target echo $\omega_o t - \phi_{T_i}$ arrive at the ith antenna element. The IF signal delivered to the PCC is $\omega t - \phi_{T_i} + \gamma_{T_i} + \delta_i$ . The phase-conjugated output is

$$\omega t + (\tfrac{1}{2})(\alpha + \delta_i - \eta_i - \mu) - [(-\phi_{T_i} + \gamma_{T_i} + \delta_i) - (\tfrac{1}{2})(\alpha + \delta_i - \eta_i - \mu)]$$

$$= \omega t + \alpha - \eta_i - \mu + \phi_{T_i} - \gamma_{T_i} \tag{45}$$

and the signal radiated in the direction of the target is

$$\omega_o t + \alpha - \mu + \phi_{T_i} \tag{46}$$

which is exactly the conjugated phase plus an arbitrary constant.

The only circuit in the system not under closed-loop control is the phase measuring and phase control branch (12) to (13) from calibrator to module. Gain and bias errors can develop in this circuit. Let the phase detector gain be in error by the factor K and let a bias M develop in its output circuit. Then the phase shift applied to the reference wave changes from (43) to

$$K[-\beta_i - \theta_i + (\tfrac{1}{2})(\alpha + \delta_i - \eta_i - \mu)] + M \tag{47}$$

The reference for the PCC becomes

$$\omega t + \beta_i + \theta_i + (47) \stackrel{\Delta}{=} \omega t + \psi . \tag{48}$$

The PCC output is

$$\omega t + 2\psi + \phi_{T_i} - \gamma_{T_i} - \delta_i \tag{49}$$

After passage through the transmitter and antenna the radiated wave toward the target is

$$\omega_o t + 2\psi + \phi_{T_i} - \delta_i + \eta_i$$

$$= \underbrace{\omega_o t + 2M + K(\alpha - \mu)}_{\text{Constant}} + \underbrace{\phi_{T_i}}_{\text{Conjugate phase}} + \underbrace{(1 - K)}_{\substack{\text{fractional}\\ \text{gain error}\\ \text{in PD}}}\underbrace{(2\beta_i + 2\theta_i + \eta_i + \delta_i)}_{\text{random variables}} \tag{50}$$

The first four terms are the conjugated phase plus an arbitrary constant. The last term represents a phase error in transmission from the ith module, the maximum effect of which is easily calculated. $\beta$ , $\theta$ , $\eta$ and $\delta$ may be assumed to be random variables independent of each other and independent of the errors in the other modules. Their sum is a random phase error. The magnitude of the net phase error $\Phi_i$ is a function of the fractional gain error K of the phase detector. Note that the phase error is not a function of the bias error of the phase detector.

Using the theory of main-lobe gain-loss referenced in Section 6 [7, Chapter 13], a tolerance can be calculated for the phase detector gain error. The random phase error is, at worst, uniformly distributed in the interval $[-\pi, \pi]$.* The variance of a uniform distribution is one-twelfth the square of the length of the interval.** Hence

$$\sigma_\Phi^2 \le \frac{(1 - K)^2 (2\pi)^2}{12} \tag{51}$$

which must not exceed 1/4 square radian to limit the loss in main-lobe gain to one dB. Taking this value as the tolerance in gain loss, the limiting allowed fractional gain error K in the phase detector is found by equating (51) to 0.25.

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*For example the probability density function of the modulo-$2\pi$ sum of two random variables, each uniformly distributed in a $2\pi$ interval, also is uniform in the interval. This case corresponds to the equality condition in (51). If the pdfs are clustered near the center of the interval, the pdf of the sum also is clustered, leading to the strict inequality in (51). The random variables in (50) will generally correspond to this case. If the pdfs are lower in the central region than at the edges, the inequality could reverse. There is no physical basis for assuming that this situation will occur in this system.

**Let $w(x) = \frac{1}{L}$, $|x| \le \frac{L}{2}$ ; = o, elsewhere. Then $\sigma_x^2 = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\, dx = \frac{L^2}{12}$

Thus

$$\frac{(1 - K)^2 (2\pi)^2}{12} = \frac{1}{4} \tag{51}$$

or K = 0.724. In other words the gain can change by 27% without causing more than a one dB loss in system performance. This is a relatively easy tolerance to maintain.

## 10. TRANSMITTERS FOR ADAPTIVE TRANSMIT-RECEIVE ARRAYS

Although most radars operate with conventional microwave sources and amplifiers such as magnetrons, klystrons and TWT's, the unusual nature of the radio camera transmitter warrants looking further afield. The solid state microwave transmitter is becoming practical and is the preferred device for the adaptive airborne system [1]. Transmitters consist of low power oscillators (which may be VCOs) driving solid state power amplifiers. Both silicon and galium arsenide devices are available. The relativistic electron beam tube is more distant. These devices are in the gyrotron family [22].

The nominal peak power levels available from these three types of microwave sources cover a remarkably large range, as given in the table below.

| MICROWAVE DEVICE | PEAK POWER |
|---|---|
| Solid state | 1 - 100 W |
| Conventional Sources | $10^5$ - $10^6$ W |
| Gyrotron | $10^8$ - $10^9$ W |

TABLE 3. Range of Peak Power available from Microwave Transmitting Devices

The choice of transmitter for the transmit-receive module depends largely on the power requirements. Consider (15) which shows that the required power per element to achieve a given power density at the target is inversely proportional to the square of the number of elements. Since the power required in radar is compatible with the (conventional) devices designed for radar ($10^5$ - $10^6$W), the power per element needed for a large transmit-receive array is $(10^5 - 10^6)/N^2$ , which is in the order of watts. Hence the solid state device is the natural component.

On the other hand when the array is organized into rigid subarrays (as in Figure 2), it is practical to use a single transmitter per subarray. If the number of subarrays with which the system is composed is small (e.g., 10) and the required peak power is large ( $\gtrsim 10^6$ W), low power conventional devices are more appropriate. Clearly the gyrotron, which is a remarkable invention, is inappropriate for a radar-range, large-array detection and/or imaging system.

## 11. CW BEACON REFERENCE SIGNAL

The phase synchronizing signal assumed in Section 5, and upon which the designs of the subsequent sections were based, was either a strong echo from a large target or a beacon response to the radar transmission. In either case, a microwave transmitter is needed. It also is possible to use a remote free-running, CW source.

The phase reference can be delivered by the means discussed in Section 8. The PCC, however, cannot be any of the circuits described in Section 6; in those circuits a time difference (of 10's to 100's of microseconds) between reception of a signal at a module and transmission from the same module provided the necessary isolation between the incoming and outgoing waves. The CW beacon, on the other hand, radiates high power continuously. Hence the synchronizing signal must be offset in frequency from the radar transmitter's radiation. The system must avoid angular displacement (squint) which normally accompanies frequency offset (see Section 7) and must also accommodate for the frequency displacement in the delivery of the phase reference.

A technique which accomplishes the PCC objectives is described by Chernoff in [8]. The synchronizing signal from the earth at frequency $\omega$ arrives at the ith element with phase

$$\phi_i(t) = \omega(t - r_i/c) \tag{53}$$

$r_i$ is the distance from the source to the ith element and c is the speed of propagation. The desired signal for transmission has conjugate phase

$$\phi_i'(t) = \omega'(t + r_i/c) + \phi_o \tag{54}$$

where $\omega' - \omega$ is the frequency offset and $\phi_o$ is a constant which must be the same at each module.

The PCC proposed by Chernoff is shown in Figure 17. The VCO is locked by the loop to frequency $\omega'$. Let its phase be $\omega' t + \phi'$. The up-link signal arrives with phase $\omega t - \phi$. Mixer 1 is an up converter, the output of which, $(\omega' + \omega)t - \phi + \phi'$ is divided in frequency and phase by two. The other input to mixer 2 is the up-link signal $\omega t + \beta$ received by some arbitrary element in the array; it constitutes the system phase reference. Mixer 2 is a down converter; its output phase is

$$\text{mixer 2 phase} = \left(\frac{\omega - \omega'}{2}\right)t + \beta + \frac{\phi - \phi'}{2} \tag{55}$$

The phase of the second input to the phase detector is $(\omega' t + \phi')/n$. The loop locks when the inputs to the phase detector are in quadrature:

$$\left[\frac{\omega}{2} - \omega'\left(\frac{1}{2} + \frac{1}{n}\right)\right]t + \beta + \frac{\phi}{2} - \phi'\left(\frac{1}{2} + \frac{1}{n}\right) = \frac{\pi}{2} \tag{56}$$

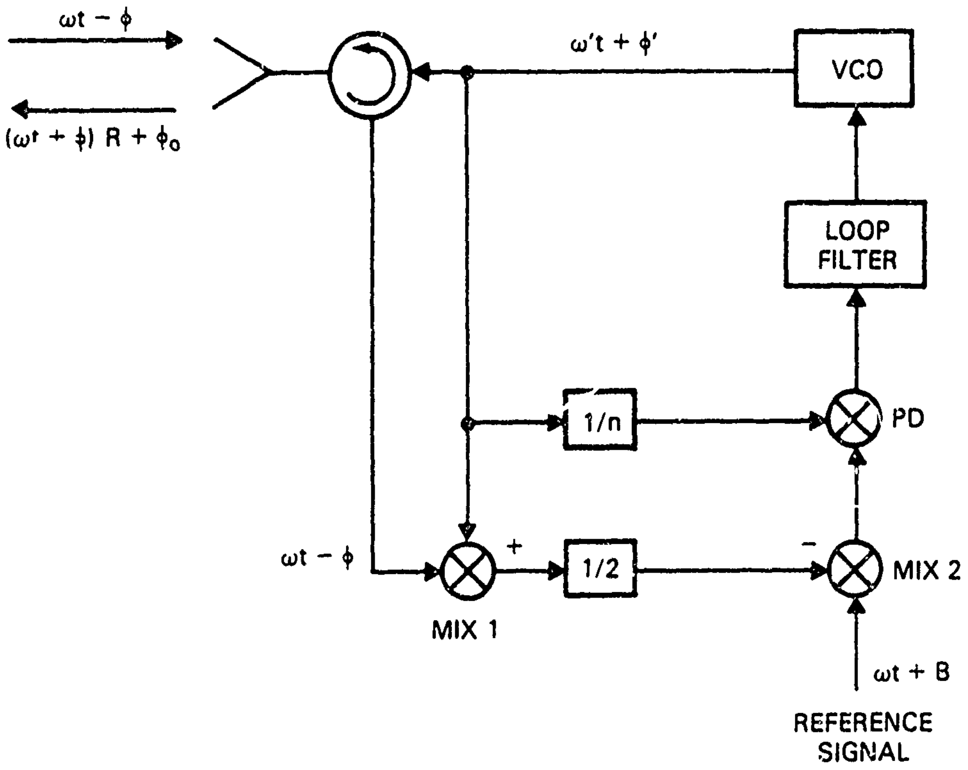


FIGURE 17. PHASE CONJUGATION CIRCUIT WITH FREQUENCY OFFSET (FROM [8])

This equation can be satisfied at all times if the first term equals zero. Thus the loop-lock conditions are

$$\omega' = \frac{\omega}{1 + \frac{2}{n}} = \omega R \tag{57}$$

and

$$\phi' = \phi R + (2\beta - \pi)R \tag{58}$$

The transmission frequency differs by a factor

$$R = \frac{1}{1 + \frac{2}{n}} \tag{59}$$

from the up-link frequency and the transmission phase is the conjugate of the received phase multiplied by the same constant, plus a constant. Thus the frequency and phase conditions of (54) are satisfied.

## 12. SUMMARY

The logical requirements for a self-adaptive, nonrigid, distributed radar antenna array are discussed. A transmitter is required to illuminate a target, the reflections from which are received by the elements in the array. The target must reradiate a nearly spherical wavefront. The phases of the received echoes are used to set the phase shifts in the antenna elements so that a receiving beam is focused on the target. The same phase information permits setting the transmission phase shifts as well.

Once the transmitting beam is formed and focused on the target the initializing illumination no longer is required. The beam is scanned by modifying the phase shifts in the same manner that is used in a conventional phased array.

Phase conjugation of the received wave at every element is necessary to achieve focused transmission. There are two primary circuit and system choices to make in the design of phase conjugating networks. The first choice is between analog and digital circuits. The second choice is between open-loop and closed-loop control of the phase shift. The bases for these choices are discussed and several circuits are given. The phase conjugating circuit at each antenna element requires a reference wave of constant frequency and fixed phase. Methods for distributing the phase reference across the array are discussed.

Coherent, focused transmission of power $P_e$ from each of N antenna elements increases the power density on a target by the factor $N^2$ relative to transmission of $P_e$ from a single element. Because of the quadratic dependence upon N, the required power per element is small, e.g., the order of watts. Hence a separate solid state transmitter per element is most appropriate.

## 13. REFERENCES


1. B. D. Steinberg and E. Jadlovker, "Distributed Airborne Array Concepts," VFRC-9-79, Valley Forge Research Center, Univ. of Pa., Phila., July 1979.

2. W. F. Gabriel, "Adaptive Arrays," Proc. IEEE, Feb. 1976.

3. Y. T. Lo, "A Mathematical Theory of Antenna Arrays with Randomly Spaced Elements," IRE Trans. Antennas Propagation, AP-12 (May 1964), 257-268.

4. B. D. Steinberg, "The Peak Sidelobe of the Phased Array having Randomly Located Elements," IEEE Trans. Antennas Propagation AP-20 (March 1972).

5. B. D. Steinberg, "On the Design of a Radio Camera for High Resolution in Microwave Imaging," J. Franklin Inst. (Dec. 1973).

6. B. D. Steinberg, "High Angular Microwave Resolution from Distorted Arrays," Proc. Symp. Soc. Photo-Optical Instrumentation Engineers, April 1980.

7. B. D. Steinberg, Principles of Aperture and Array System Design, John Wiley, 1976.

8. R. Chernoff, "Large Active Arrays for Space Applications," IEEE Trans. Antennas Propagation, AP-27 (July 1979) 489-96.

9. E. D. Sharp and M. A. Diab, "Van Atta Reflector Array," IRE Trans. Antennas Propagation,AP-8 (July 1960) 436-438.

10. R. C. Hansen, "Communications Satellites using Arrays," Proc. IRE, vol. 49 (June 1961) 1066-1074.

11. F. M. Gardner, Phaselock Techniques, 2nd Edition, John Wiley, New York, 1979.

12. A. J. Viterbi, Principles of Coherent Communication, McGraw-Hill, New York, 1966.

13. W. C. Lindsey, Synchronization Systems in Communications and Control, Prentice Hall, Englewood Cliffs, New Jersey, 1972.

14. J. Klapper and J. T. Frankle, Phase-locked and Frequency Feedback Systems: Principles and Techniques, Academic Press, 1972.

15. W. C. Lindsey and R. C. Tausworthe, "Bibliography of the Theory and Application of the Phase-lock Principle," Tech. Rep. 32-1581, Jet Propulsion Lab., Calif. Inst. of Technology, Pasadena, Calif., Apr. 1973.

16. E. Grimberg and Y. Bar-Ness, "Acquisition Behavior of Pulse Tracking Phase-Locked Loop," Int'l. J. Electronics, Vol. 47 (July 1979), 1-15.

17. B. D. Steinberg, "Radar Feedback Filters," Chapt. 3, Part 4 of Modern Radar, R. S. Berkowitz, editor, John Wiley, New York, 1965, pp 506-511.

18. P. Glazer, "Power from the Sun: Its Future," Science, Vol. 162 (Nov. 22, 1968), 857-861.

19. "Satellite Power System - Concept Development and Evaluation Program," USDOE and NASA Report No. DOE/ER-0023. Available from Nat'l Tech. Info. Services, U.S. Dept. of Commerce, Jan 1979.

20. W. C. Lindsey and A. V. Kantek, "Automatic Phase Control in Solar Power Satellite Systems," Prepared for NASA/JSC, TR-7802-0977, Feb. 1978, LinCom Corp., P.O. Box 2793D, Pasadena, CA.

21. B. D. Steinberg and E. N. Powers, "High Angular Resolution Radar," Proc. IEEE Int'l. Radar Symposium, Arlington, VA April 1975.

22. T. F. Godlove and V. L. Granatstein, "Relativistic Electron Beam Sources," Soc. Photo-Optical Instrumentation Engineers, Vol. 105, Far Infrared/Submillimeter Wave Technology/Applications (1977).